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Teleparallel Energy-Momentum Distribution of Spatially Homogeneous Rotating Spacetimes

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Abstract The energy-momentum distribution of spatially homogeneous rotating spacetimes in the context of teleparallel theory of gravity is investigated. For this purpose, we use the teleparallel version of Möller prescription. It is found that the components of energy-momentum density are finite and well-defined but are different from General Relativity. However, the energy-momentum density components become the same in both theories under certain assumptions. We also analyse these quantities for some special solutions of the spatially homogeneous rotating spacetimes.

Keywords Teleparallel theory · Energy

1 Introduction

Till now, several attempts have been made to unify gravitation with other interactions, including Einstein's, which led to the investigation of basic structures of gravitation other than the metric tensor. These structures yield the metric tensor as a by product. Tetrad field is one of these structures which leads to the theory of teleparallel gravity (TPG) [1, 2]. TPG is an alternative theory of gravity which corresponds to a gauge theory of translation group [3, 4] based on Weitzenböck geometry [5]. This theory is characterized by the vanishing of curvature identically while the torsion is taken to be non-zero. In TPG, gravitation is attributed to torsion [4] which plays a role of force [6]. In General Relativity (GR), gravitation geometrizes the underlying spacetime. The translational gauge potentials appear as a nontrivial part of the tetrad field and induce a teleparallel (TP) structure on spacetime which is directly related to the presence of a gravitational field. In some other theories [3–8], torsion is only relevant when spins are important [9]. This point of view indicates that torsion might represent additional degrees of freedom as compared to curvature. As a result, some new

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physics may be associated with it. Teleparallelism is naturally formulated by gauging external (spacetime) translations which are closely related to the group of general coordinate transformations underlying GR. Thus the energy-momentum tensor represents the matter source in the field equations of tetradic theories of gravity like in GR.

The localization of energy (i.e., to express it as a unique tensor quantity) has been a longstanding, open and controversial problem in GR [10] which is still with out a definite answer. As a pioneer, Einstein [11] introduced the energy-momentum pseudo-tensor and then Landau-Lifshitz [12], Papapetrou [13], Bergmann [14], Tolman [15] and Weinberg [16] proposed their own prescriptions to resolve this issue. All these prescriptions can provide meaningful results only in Cartesian coordinates. But Möller [17] introduced a coordinate-independent prescription. The idea of coordinate-independent quasi-local mass was introduced [18] by associating a Hamiltonian term to each gravitational energy-momentum pseudo-tensor. Later, a Hamiltonian approach in the frame of Schwinger condition [19] was developed, followed by the construction of a Lagrangian density of TP equivalent to GR [4, 6, 20, 21]. Many authors explored many examples in the framework of GR and found that different energy-momentum complexes can give either the same [22–27] or different [28–33] results for a given spacetime.

Mikhail et al. [34] defined the supperpotential in the Möller's tetrad theory which has been used to find the energy in TPG. Vargas [35] defined the TP version of Bergman, Einstein and Landau-Lifshitz prescriptions and found that the total energy of the closed Friedman-Robinson-Walker universe is zero by using the last two prescriptions. This agrees with the results of GR available in literature [36–38]. Later, many authors [39–45] used TP version of these prescriptions and showed that energy may be localized in TPG. Keeping this point in mind, this paper is addressed to investigate the energy-momentum distribution of spatially homogeneous and rotating spacetimes by using the TP version of Möller prescription.

This paper is organized as follows. Section 2 contains an overview of the TP theory. In Sect. 3, we investigate the energy-momentum distribution of spatially homogeneous rotating spacetimes by using the TP version of Möller prescription. Section 4 is devoted to discuss five special cases of the metric representing spatially homogeneous rotating spacetimes. The last section will furnish a summary and a discussion of the results obtained.

2 An Overview of the Teleparallel Theory

The TP theory is based on Weitzenböck connection given as [46]

$$\Gamma^{\theta}{}_{\mu\nu} = h_a{}^{\theta}\partial_\nu h^a{}_\mu, \tag{1}$$

where $h_a{}^\nu$ is a non-trivial tetrad. Its inverse field is denoted by $h^a{}_\mu$ and satisfying the relations

$$h^a{}_\mu h_a{}^\nu = \delta_\mu{}^\nu; \qquad h^a{}_\mu h_b{}^\mu = \delta^a{}_b. \tag{2}$$

In this paper, the Latin alphabet $(a, b, c, \ldots = 0, 1, 2, 3)$ will be used to denote tangent space indices and the Greek alphabet $(\mu, \nu, \rho, \ldots = 0, 1, 2, 3)$ to denote spacetime indices. The Riemannian metric in TP theory arises as a by product [4] of the tetrad field given by

$$g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu, \tag{3}$$

where η_{ab} is the Minkowski metric $\eta_{ab} = \mathrm{diag}(+1, -1, -1, -1)$. For the Weitzenböck spacetime, the torsion is defined as [2]

$$T^{\theta}{}_{\mu\nu} = \Gamma^{\theta}{}_{\nu\mu} - \Gamma^{\theta}{}_{\mu\nu} \tag{4}$$

which is antisymmetric w.r.t. its last two indices. Due to the requirement of absolute parallelism, the curvature of the Weitzenböck connection vanishes identically. The Weitzenböck connection also satisfies the relation

$$\Gamma^{0\theta}{}_{\mu\nu} = \Gamma^{\theta}{}_{\mu\nu} - K^{\theta}{}_{\mu\nu}, \tag{5}$$

where

$$K^{\theta}{}_{\mu\nu} = \frac{1}{2}[T_{\mu}{}^{\theta}{}_{\nu} + T_{\nu}{}^{\theta}{}_{\mu} - T^{\theta}{}_{\mu\nu}] \tag{6}$$

is the *contortion tensor* and $\Gamma^{0\theta}{}_{\mu\nu}$ are the Christoffel symbols in GR.

Mikhail et al. [34] defined the super-potential of the Möller tetrad theory as

$$U_{\mu}{}^{\nu\beta} = \frac{\sqrt{-g}}{2\kappa} P^{\tau\nu\beta}_{\chi\rho\sigma}[V^{\rho} g^{\sigma\chi} g_{\mu\tau} - \lambda g_{\tau\mu} K^{\chi\rho\sigma} - (1-2\lambda) g_{\tau\mu} K^{\sigma\rho\chi}], \tag{7}$$

where

$$P^{\tau\nu\beta}_{\chi\rho\sigma} = \delta_{\chi}{}^{\tau} g^{\nu\beta}_{\rho\sigma} + \delta_{\rho}{}^{\tau} g^{\nu\beta}_{\sigma\chi} - \delta_{\sigma}{}^{\tau} g^{\nu\beta}_{\chi\rho} \tag{8}$$

and $g^{\nu\beta}_{\rho\sigma}$ is a tensor quantity defined by

$$g^{\nu\beta}_{\rho\sigma} = \delta_{\rho}{}^{\nu}\delta_{\sigma}{}^{\beta} - \delta_{\sigma}{}^{\nu}\delta_{\rho}{}^{\beta}. \tag{9}$$

$K^{\sigma\rho\chi}$ is the contortion tensor given by (6), g is the determinant of the metric tensor $g_{\mu\nu}$, λ is the free dimensionless coupling constant of TPG, κ is the Einstein constant and V_{μ} is the basic vector field given by

$$V_{\mu} = T^{\nu}{}_{\nu\mu}. \tag{10}$$

The energy-momentum density is defined as

$$\Xi^{\nu}_{\mu} = U^{\nu\rho}_{\mu}{}_{,\rho}, \tag{11}$$

where comma means ordinary differentiation. The momentum 4-vector of Möller prescription can be expressed as

$$P_{\mu} = \int_{\Sigma} \Xi^{0}_{\mu} dx dy dz, \tag{12}$$

where P_0 gives the energy and P_1, P_2 and P_3 are the momentum components while the integration is taken over the hyper-surface element Σ described by $x^0 = t = constant$. The energy may be given in the form of surface integral [17] as

$$E = \lim_{r\to\infty} \int_{r=constant} U_0{}^{0\rho} u_{\rho} dS, \tag{13}$$

where u_{ρ} is the unit three-vector normal to the surface element dS.

3 Teleparallel Energy of Spatially Homogeneous Rotating Spacetimes

After the pioneering work of Gamow [47] and Gödel [48], the idea of global rotation of the universe has become a favorite topic in the research field of GR. The metric representing spatially homogeneous universes with rotation but no shear can be given as

$$ds^2 = dt^2 - dr^2 - A(r)d\phi^2 - dz^2 + 2B(r)dtd\phi, \tag{14}$$

where $A(r)$ and $B = B(r)$ are arbitrary functions. The metric given by (14) represents five spacetimes [48, 49], which can be achieved by choosing particular values of the metric functions A and B. Using the procedure adopted in the papers [50, 51], the tetrad components of the above metric can written as

$$h^a{}_\mu = \begin{bmatrix} 1 & 0 & B & 0 \\ 0 & \cos\phi & -\Delta\sin\phi & 0 \\ 0 & \sin\phi & \Delta\cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

with its inverse

$$h_a{}^\mu = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{B}{\Delta}\sin\phi & \cos\phi & -\frac{1}{\Delta}\sin\phi & 0 \\ -\frac{B}{\Delta}\cos\phi & \sin\phi & \frac{1}{\Delta}\cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{16}$$

Here $\Delta = \Delta(r) = \sqrt{A + B^2}$. In view of (15) and (16), (1) yields the following non-vanishing components of the Weitzenböck connection:

$$\begin{aligned} &\Gamma^0{}_{12} = -\frac{B}{\Delta}, \qquad \Gamma^0{}_{21} = B' - \frac{B\Delta'}{\Delta}, \\ &\Gamma^2{}_{12} = \frac{1}{\Delta}, \qquad \Gamma^2{}_{21} = \frac{\Delta'}{\Delta}, \\ &\Gamma^1{}_{22} = -\Delta. \end{aligned} \tag{17}$$

The corresponding non-vanishing components of the torsion tensor are

$$\begin{aligned} &T^0{}_{12} = B' + \frac{B}{\Delta}(1 - \Delta'), \\ &T^2{}_{12} = \frac{1}{\Delta}(\Delta' - 1). \end{aligned} \tag{18}$$

Substituting these values in (10) and then multiplying by g^{11} and g^{33} respectively, we get

$$V^1 = \frac{1}{\Delta}(\Delta' - 1), \tag{19}$$

$$V^3 = 0. \tag{20}$$

In view of (18) and (6), the non-vanishing components of the contorsion tensor are

$$\begin{aligned} K^{100} &= \frac{B}{\Delta^3}(B + B'\Delta - B\Delta') = -K^{010}, \\ K^{122} &= \frac{1}{\Delta^3}(\Delta' - 1) = -K^{212}, \\ K^{102} &= K^{120} = \frac{B}{\Delta^3}(1 - \Delta') + \frac{B'}{2\Delta^2} = -K^{012} = -K^{210}, \\ K^{021} &= \frac{B'}{2\Delta^2} = -K^{201}. \end{aligned} \tag{21}$$

It should be mentioned here that the contorsion tensor is antisymmetric w.r.t. its first two indices. Making use of (19–21) in (7), we obtain the required independent non-vanishing components of the supperpotential in Möller's tetrad theory as

$$\begin{aligned} U_0^{01} &= \frac{1}{2\kappa\Delta}\{(1+\lambda)BB' + 2\Delta(1-\Delta')\} = U_0^{10}, \\ U_2^{01} &= \frac{1}{2\kappa\Delta}\{(1+\lambda)B^2B' + (1-\lambda)\Delta^2B' + 2B\Delta(1-\Delta')\} = -U_2^{10}, \\ U_0^{21} &= -\frac{1}{2\kappa\Delta}(1+\lambda)B' = -U_0^{12}. \end{aligned} \tag{22}$$

It is worth mentioning here that the supperpotential is skew symmetric w.r.t. its last two indices. When we make use of (22) in (11), the energy density turns out to be

$$\Xi_0^0 = \frac{1}{2\kappa\Delta^2}\{(1+\lambda)(\Delta BB'' + \Delta B'^2 - BB'\Delta') - 2\Delta^2\Delta''\}. \tag{23}$$

For $\lambda = 1$

$$E^d{}_{\mathrm{TPT}} = E^d{}_{\mathrm{GR}} - \frac{\Delta''}{\kappa}, \tag{24}$$

where E^d stands for energy density. It is clear that the energy density in both the theories is same in the case, if possible, $\Delta' = constant$. The only non-zero component of momentum density is along ϕ direction and (for $\lambda = 1$) is given by

$$\Xi_2^0 = \frac{1}{\kappa\Delta^2}\{\Delta(AB'' - A''B) - \Delta'(AB' - A'B)\} - \frac{1}{\kappa}(B''\Delta - 2B'\Delta') \tag{25}$$

that is,

$$M^d{}_{\mathrm{TPT}} = M^d{}_{\mathrm{GR}} - \frac{1}{\kappa}(B''\Delta - 2B'\Delta'), \tag{26}$$

where M^d stands for momentum density. Again in the case, if possible, when B is constant the momentum density in TPT and GR become same.

4 Some Special Cases of Spatially Homogeneous Rotating Spacetimes

In this section, we would like to mention some special cases of spatially homogeneous rotating spacetimes. At the end, the results for these cases will be given in tables.

Table 1 Energy densities of the reduced cases of spatially homogeneous rotating spacetimes

Spacetime	In GR [64]	In TPT	Remarks
Rb	$\Theta^0_0=\frac{2s^*}{\pi}$	$\Xi^0_0=\frac{3s^*}{2\pi}$	$\Xi^0_0=\frac{3}{4}\Theta^0_0$
SR	$\Theta^0_0=\frac{r}{2\pi}$	$\Xi^0_0=\frac{r}{2\pi}$	$\Xi^0_0=\Theta^0_0$
HV	$\Theta^0_0=\frac{s}{4\sqrt{2}\pi}$	$\Xi^0_0=\frac{s}{8\sqrt{2}\pi}$	$\Xi^0_0=\frac{1}{2}\Theta^0_0$
GF	$\Theta^0_0=\frac{cs}{\pi}$	$\Xi^0_0=\frac{cs}{2\pi}$	$\Xi^0_0=\frac{1}{2}\Theta^0_0$
SG	$\Theta^0_0=\frac{a^2}{4\sqrt{2}\pi}e^{ar}$	$\Xi^0_0=\frac{a^2}{8\pi}e^{ar}$	$\Xi^0_0=\frac{1}{\sqrt{2}}\Theta^0_0$

1. *The Reboucas (Rb) spacetime* [49]
By choosing $A(r)=-(1+3c^{*2})$ and $B(r)=2c^*$, where $c^*=\cosh 2r$ and $s^*=\sinh 2r$, the metric (14) takes the form

$$ds^2=dt^2-dr^2+(1+3c^{*2})d\phi^2-dz^2+4c^*dtd\phi. \tag{27}$$

2. *The Som-Raychaudhuri (SR) spacetime* [49]
By choosing $A(r)=r^2(1-r^2)$ and $B(r)=r^2$, the metric (14) reduces to

$$ds^2=dt^2-dr^2-r^2(1-r^2)d\phi^2-dz^2+2r^2dtd\phi. \tag{28}$$

3. *The Hoenselaers-Vishveshwara (HV) spacetime* [49]
By choosing $A(r)=-\frac{1}{2}(c-1)(c-3)$ and $B(r)=c-1$, where $c=\cosh r$, the metric (14) implies that

$$ds^2=dt^2-dr^2+\frac{1}{2}(c-1)(c-3)d\phi^2-dz^2+2(c-1)dtd\phi. \tag{29}$$

4. *The Gödel-Friedmann (GF) spacetime* [49]
By choosing $A(r)=s^2(1-s^2)$ and $B(r)=\sqrt{2}s^2$, where $s=\sinh r$, the metric (14) turns out as

$$ds^2=dt^2-dr^2-s^2(1-s^2)d\phi^2-dz^2+2\sqrt{2}s^2dtd\phi. \tag{30}$$

5. *The Stationary Gödel (SG) spacetime* [48]
By choosing $A(r)=-\frac{1}{2}e^{2ar}$ and $B(r)=e^{ar}$, where $a=constant$, the metric (14) becomes

$$ds^2=dt^2-dr^2+\frac{1}{2}e^{2ar}d\phi^2-dz^2+2e^{ar}dtd\phi. \tag{31}$$

The corresponding energy-momentum densities of these spacetimes are given in Tables 1 and 2.

5 Summary and Discussion

There is a large literature available [52–60] about the study of TP versions of the exact solutions of GR. Recently, Pereira et al. [50] obtained the TP versions of the Schwarzschild and the stationary axisymmetric Kerr solutions of GR. They proved that the axial-vector torsion plays the role of the gravitomagnetic component of the gravitational field in the case of slow rotation and weak field approximations. In previous papers [51, 61–63], we have found the

Table 2 Momentum densities of the reduced cases of spatially homogeneous rotating spacetimes

Spacetime	In GR [64]	In TPT	Remarks
Rb	$\Theta^0_2 = \frac{6c^*s^*}{\pi}$	$\Xi^0_2 = \frac{5s^*c^*}{\pi}$	$\Xi^0_2 = \frac{5}{6}\Theta^0_2$
SR	$\Theta^0_2 = \frac{r^3}{\pi}$	$\Xi^0_2 = \frac{4r^3+r}{4\pi}$	$\Xi^0_2 = \Theta^0_2 + \frac{r}{4\pi}$
HV	$\Theta^0_2 = \frac{s(c-1)}{4\sqrt{2}\pi}$	$\Xi^0_2 = \frac{s(3c-2)}{8\sqrt{2}\pi}$	$\Xi^0_2 = \Theta^0_2 + \frac{cs}{8\sqrt{2}\pi}$
GF	$\Theta^0_2 = \frac{\sqrt{2}cs^3}{\pi}$	$\Xi^0_2 = \frac{cs(5s^2+c^2)}{2\sqrt{2}\pi}$	$\Xi^0_2 = \frac{5}{4}\Theta^0_2 + \frac{c^3s}{2\sqrt{2}\pi}$
SG	$\Theta^0_0 = \frac{a^2}{4\sqrt{2}\pi}e^{2ar}$	$\Xi^0_2 = \frac{3a^2}{8\sqrt{2}\pi}e^{2ar}$	$\Xi^0_2 = \frac{3}{2}\Theta^0_2$

TP versions of the Friedmann models, Lewis-Papapetrou spacetimes, stationary axisymmetric Einstein-Maxwell solutions and also discussed the energy-momentum distribution in last three papers.

The problem of localization of energy has been reconsidered, in the frame work of TPG, by many scientists. Some authors [35, 39–45] showed that energy-momentum can also be localized in this theory. It has been shown that the results of the two theories are either same or disagree with each other for a given spacetime. Möller showed that a tetrad description of a gravitational field equation allows a more satisfactory treatment of the energy-momentum complex than does GR.

Currently [61–63], we considered some particular spacetimes and calculated the energy-momentum densities and found that the results disagree in general but can coincide under certain conditions. In this paper, we extended the work and explore the energy-momentum distribution of spatially homogeneous rotating spacetimes by using the TP version of Möller's prescription. We found that the results are not generally same as found in the context of GR [64]. If, possibly, we choose $\Delta' = costant$ (as in the case of SR spacetime), then

$$E^d{}_{\mathrm{TPG}} = E^d{}_{\mathrm{GR}} \tag{32}$$

and for $B = constant$

$$M^d{}_{\mathrm{TPG}} = M^d{}_{\mathrm{GR}}. \tag{33}$$

It should be worth mentioning here that only the non-vanishing component of the momentum density is along ϕ-direction which is due the cross term $dtd\phi$ involving in the metric (14). It is similar to the case of stationary axisymmetric Einstein-Maxwell solutions [61]. Further, we consider some special cases and found their energy-momentum distribution by replacing the corresponding values of the metric functions. It is shown that the energy density is same in both the theories only in the case of SR spacetime while for the other cases it turns out as multiple of some real number, as given in Table 1. The relation between the components of momentum density in GR [64] and TPT are given in last column of Table 2 for every reduced spacetime.

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References

1. Müller-Hoisson, F., Nitsch, J.: Phys. Rev. D **28**, 718 (1983)

2. De Andrade, V.C., Pereira, J.G.: Gen. Relativ. Gravit. **30**, 263 (1998)
3. Hehl, F.W., McCrea, J.D., Mielke, E.W., Ne'emann, Y.: Phys. Rep. **258**, 1 (1995)
4. Hayashi, K., Shirafuji, T.: Phys. Rev. D **19**, 3524 (1979)
5. Weitzenböck, R.: Invarianten Theorie. Noordhoff, Groningen (1923)
6. De Andrade, V.C., Pereira, J.G.: Phys. Rev. D **56**, 4689 (1997)
7. Blagojecvic, M.: Gravitation and Gauge Symmetries. IOP Publishing, Bristol (2002)
8. Hammond, R.T.: Rep. Prog. Phys. **65**, 599 (2002)
9. Gronwald, F., Hehl, F.W.: On the gauge aspects of gravity. In: Bergmann, P.G., et al. (eds.) Proceedings of the 14th School of Cosmology and Gravitation, Erice, Italy. World Scientific, Singapore (1996)
10. Misner, C.W., Thorne, K.S., Wheeler, J.A.: Gravitation. Freeman, New York (1973)
11. Einstein, A.: Sitzungsber. Preus. Akad. Wiss. Berlin (Math. Phys.) **778** (1915). Addendum Sitzungsber. Preus. Akad. Wiss. Berlin (Math. Phys.) **779** (1915)
12. Landau, L.D., Lifshitz, E.M.: The Classical Theory of Fields. Addison-Wesley, Reading (1962)
13. Papapetrou, A.: Proc. R. Ir. Acad. A **52**, 11 (1948)
14. Bergman, P.G., Thomson, R.: Phys. Rev. **89**, 400 (1958)
15. Tolman, R.C.: Relativity, Thermodynamics and Cosmology. Oxford University Press, Oxford (1934)
16. Weinberg, S.: Gravitation and Cosmology. Wiley, New York (1972)
17. Möller, C.: Ann. Phys. (NY) **4**, 347 (1958)
18. Chang, C.C., Nester, J.M.: Phys. Rev. Lett. **83**, 1897 (1999) and references therein
19. Schwinger, J.: Phys. Rev. **130**, 1253 (1963)
20. De Andrade, V.L., Guillen, L.C.T., Pereira, J.G.: Phys. Rev. Lett. **84**, 4533 (2000)
21. Aldrovendi, R., Pereira, J.G.: An introduction to gravitation theory. Preprint
22. Virbhadra, K.S.: Phys. Rev. D **60**, 104041 (1999)
23. Virbhadra, K.S.: Phys. Rev. D **42**, 2919 (1990)
24. Virbhadra, K.S.: Phys. Lett. B **331**, 302 (1994)
25. Virbhadra, K.S., Parikh, J.C.: Phys. Lett. B **317**, 312 (1993)
26. Rosen, N., Virbhadra, K.S.: Gen. Relativ. Gravit. **25**, 429 (1993)
27. Xulu, S.S.: Astrophys. Space Sci. **283**, 23 (2003)
28. Sharif, M.: Int. J. Mod. Phys. A **17**, 1175 (2002)
29. Sharif, M.: Int. J. Mod. Phys. A **18**, 4361 (2003)
30. Sharif, M.: Int. J. Mod. Phys. A **19**, 1495 (2004)
31. Sharif, M.: Int. J. Mod. Phys. D **13**, 1019 (2004)
32. Sharif, M., Fatima, T.: Nuovo Cim. B **120**, 533 (2005)
33. Sharif, M., Fatima, T.: Int. J. Mod. Phys. A **20**, 4309 (2005)
34. Mikhail, F.I., Wanas, M.I., Hindawi, A., Lashin, E.I.: Int. J. Theor. Phys. **32**, 1627 (1993)
35. Vargas, T.: Gen. Relativ. Gravit. **36**, 1255 (2004)
36. Penrose, R.: Proc. R. Soc. Lond. A **381**, 53 (1982)
37. Tod, K.P.: Proc. R. Soc. Lond. A **388**, 457 (1983)
38. Rosen, N.: Gen. Relativ. Gravit. **26**, 323 (1994)
39. Nashed, G.G.L.: Nuovo Cim. B **119**, 967 (2004)
40. Salti, M., Havare, A.: Int. J. Mod. Phys. A **20**, 2169 (2005)
41. Salti, M.: Int. J. Mod. Phys. A **20**, 2175 (2005)
42. Salti, M.: Astrophys. Space Sci. **229**, 159 (2005)
43. Aydogdu, O., Salti, M.: Astrophys. Space Sci. **229**, 227 (2005)
44. Aydogdu, O., Salti, M., Korunur, M.: Acta Phys. Slovaca **55**, 537 (2005)
45. Sezgin, A., Melis, A., Tarhan, I.: Acta Physica Polonica B (2007, to appaer)
46. De Andrade, V.L., Guillen, L.C.T., Pereira, J.G.: An Introduction to Geometrical Physics. World Scientific, Singapore (1995)
47. Gamow, G.: Nature (Lond.) **158**, 549 (1946)
48. Gödel, K.: Rev. Mod. Phys. **21**, 447 (1949)
49. Krori, K.D., Borgohain, P., Das (Kar), D.: J. Math. Phys. **29**, 1645 (1988)
50. Pereira, J.G., Vargas, T., Zhang, C.M.: Class. Quantum Gravity **18**, 833 (2001)
51. Sharif, M., Amir, M.J.: Gen. Relativ. Gravit. **38**, 1735 (2006)
52. Hehl, F.W., Macias, A.: Int. J. Mod. Phys. D **8**, 399 (1999)
53. Obukhov, Y.N., Vlachynsky, E.J., Esser, W., Tresguerres, R., Hehl, F.W.: Phys. Lett. A **220**, 1 (1996)
54. Baekler, P., Gurses, M., Hehl, F.W., McCrea, J.D.: Phys. Lett. A **128**, 245 (1988)
55. Vlachynsky, E.J., Esser, W., Tresguerres, R., Hehl, F.W.: Class. Quantum Gravity **13**, 3253 (1996)
56. Ho, J.K., Chern, D.C., Nester, J.M.: Chin. J. Phys. **35**, 640 (1997)
57. Hehl, F.W., Lord, E.A., Smally, L.L.: Gen. Relativ. Gravit. **13**, 1037 (1981)
58. Kawa, T., Toma, N.: Prog. Theor. Phys. **87**, 583 (1992)
59. Nashed, G.G.L.: Phys. Rev. D **66**, 060415 (2002)

60. Nashed, G.G.L.: Gen. Relativ. Gravit. **34**, 1074 (2002)
61. Sharif, M., Amir, M.J.: Gen. Relativ. Gravit. **39**, 989 (2007)
62. Sharif, M., Amir, M.J.: Mod. Phys. Lett. A **22**, 425 (2007)
63. Sharif, M., Amir, M.J.: Mod. Phys. Lett. A (2007, to appear)
64. Havare, A., Salti, M., Yetkin, T.: On the energy-momentum densities of the cylindrically symmetric gravitational waves. gr-qc/0502057